

Breaking *CPT* by mixed noncommutativity

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The mixed component of the noncommutative parameter $\theta_{\mu M}$, where $\mu=0,1,2,3$ and M is an extra dimensional index, may violate four-dimensional *CPT* invariance. We calculate one- and two-loop induced couplings of $\theta_{\mu 5}$ with the four-dimensional axial vector current and with the *CPT* odd $\text{dim}=6$ operators starting from five-dimensional Yukawa and $U(1)$ theories. The resulting bounds from clock comparison experiments place a stringent constraint on $\theta_{\mu 5}$, $|\theta_{\mu 5}|^{-1/2} \gtrsim 5 \times 10^{11}$ GeV. Orbifold projection and/or localization of fermions on a 3-brane lead to *CPT*-conserving physics, in which case the constraints on $\theta_{\mu 5}$ are softened.

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I. INTRODUCTION

Non-commutative field theories and their realizations in string theory have been a subject of intensive theoretical research over the past few years (see e.g. [1,2]). Much of this excitement has gone totally unnoticed by particle phenomenology for the following simple reason. The presence of the antisymmetric tensor $\theta_{\mu\nu}$ as a constant background violates Lorentz invariance, a possibility excluded to an impressive accuracy by various low-energy precision measurements (see e.g. [3]). In our previous paper, Ref. [4], we have shown that in the low-energy effective interaction linearized in the non-commutative parameter, $\theta_{\mu\nu}$ couples to the nucleon spin, $\bar{N}\sigma_{\mu\nu}N$, with strength proportional to the cube of the characteristic hadronic scale, $\Lambda_{\text{had}} \sim 1$ GeV. This analysis has been done at the tree level in order to avoid potential problems with the issues of renormalizability of the noncommutative theories, the necessity to introduce a cutoff, etc. This coupling generates an additional, magnetic-field-independent, contribution to the nucleon Larmor frequency. Therefore, this interaction has the signature of a constant magnetic field of a fixed direction and can be searched through a precise monitoring of sidereal variation of the magnetic field. Reference [5] places the limit on the possible size of such an interaction at the level of $\nu = 100$ nHz. Comparing it with the result of the theoretical calculation in [4], one arrives at an incredibly strong constraint, $\Lambda_{NC} = 1/\sqrt{\theta} \gtrsim 5 \times 10^{14}$ GeV. If non-commutativity is realized somehow only in the leptonic sector, the $\theta_{\mu\nu} \bar{N}\sigma_{\mu\nu}N$ interaction is still generated, although with $(\alpha/\pi)^2$ suppression compared to the non-commutative QCD case. This relaxes the limit down to the 10^{12} GeV level. Later it was argued in Refs. [6,7] that $\sigma_{\mu\nu}\theta_{\mu\nu}$ operator can be generated at the loop level

with a quadratically divergent integral which may bring even tighter bounds which scale with the cutoff.

Subsequent analyses have addressed the possibility to observe $\theta_{\mu\nu}$ in future collider experiments [8] and in the neutral K and B meson systems [9]. If the former cannot do much better than $\Lambda_{NC} \sim 1$ TeV, the neutral kaons could in principle be quite sensitive to Λ_{NC} . However, even with the most favorable assumptions that the non-commutativity generates somehow an effective $\Delta F=2$ transition $\theta(dOs) \times (\bar{d}Os)$ of unsuppressed strength, one can get only to the level $\Lambda_{NC} \sim 10^9$ GeV. This is in the range already excluded by the clock comparison experiments. Generically, any attempt to construct a fully non-commutative standard model (e.g. Ref. [10]) will have to comply with the bound obtained in [4] which would make $\theta_{\mu\nu}$ totally unobservable for conventional particle physics experiments. (A possible exception could be measurements of the refraction index over cosmological distances [11] or the search for the imprints of non-commutative inflation [12], when the characteristic energy scales could be quite high and compensate for the extreme smallness of Λ_{NC}^{-2} .) Other relevant works on the observational consequences of non-commutativity include [13].

Here we study phenomenological consequences of mixed non-commutativity $\theta_{\mu M}$, where μ is a normal 4D index and M is along extra dimensions. The presence of such a component is perceived by a 4D observer as a constant 4-vector, which obviously breaks Lorentz invariance. The purpose of this work is to show that in certain classes of models this background may also break 4D *CPT* invariance. Indeed, the transformation of $\theta_{\mu M}$ under the *CPT* reflection is similar to the behavior of charge times the $U(1)$ field strength, $eF_{\mu M}$, for which we know that $CPT(eF_{\mu M}) = CPT(e\partial/\partial x^M A_\mu) = -1$. Note that parity is defined here in a 4D sense and thus $P(\partial/\partial x^M) = 1$.

Unlike the previous case with the breaking of Lorentz invariance by $\theta_{\mu\nu}$ (in which case *CPT* is preserved [14]) for which no plausible low-energy physics motivation exists, one can think of baryogenesis-motivated reasons to study the

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CPT noninvariant interactions. Namely, we refer to an interesting idea [15] that the breaking of *CPT* effectively comprises two out of three Sakharov's conditions needed for baryogenesis, requiring only the breaking of the baryon number as an extra ingredient. If baryogenesis happens at the early cosmological epoch with characteristic energy scales $E^4 \sim 1/(\theta_{\mu M})^2$, a strong power-like suppression by Λ_{NC} may be lifted. Another interesting idea is the *CPT*-odd terms in the neutrino sector [16] which help to reconcile different neutrino anomalies.

We study the possibility of *CPT* violation by $\theta_{\mu 5}$ and find that it is possible in a 5-dimensional model with Dirac fermions and scalar particles and/or gauge bosons residing in all 5 dimensions. After the compactification of the fifth dimension, the zero level Kaluza-Klein mode of the fermion acquires the coupling with $\theta_{\mu 5}$ through the axial-vector current or the *CPT* non-invariant dim=6 operators. Specifying this to the case of QED, we effectively get $\theta_{\mu 5} \partial_\mu F_{\alpha\beta} \bar{N} \sigma^{\alpha\beta} \gamma_5 N$ which results in strong bounds on $\theta_{\mu 5}$. Such a model could arise in a string theory scenario. One starts with a D4 brane in type IIA string theory with non-vanishing Neveu-Schwarz–Neveu-Schwarz (NS-NS) 2-form along the world volume ($B_{0i}=0$ for unitarity reasons) and take the Seiberg-Witten limit. The result is a non-commutative (NC) 5-dimensional theory. All fields confined on the brane are free to propagate in 5 dimensions. Then one further compactifies one dimension. On the other hand, we show that in the models with intrinsically 4D fermions, e.g. localized on a (3+1)-dimensional domain wall, the *CPT* may be conserved and only even powers of $\theta_{\mu 5}$ may appear in the low-energy effective Lagrangian.

II. 5D MODEL WITH LOW-ENERGY 4D *CPT* VIOLATION

We begin by recalling that the non-vanishing commutation relations

$$[\hat{x}^a, \hat{x}^b] = i\theta^{ab}, \quad [\theta^{ab}, \hat{x}^c] = 0 \quad (1)$$

in coordinate space ($a, b, c = 0, 1, 2, 3, M_1, \dots, M_p$) lead to a modification of the interaction terms in the field theory through the Moyal product, given by

$$\begin{aligned} \phi_1 * \phi_2(x) = \exp\left(i \frac{1}{2} \theta^{\mu\nu} \frac{\partial}{\partial \xi^\mu} \frac{\partial}{\partial \zeta^\nu}\right) \phi_1(x + \xi) \\ \times \phi_2(x + \zeta) \Big|_{\xi=\zeta=0}. \end{aligned} \quad (2)$$

This type of commutation relations can arise in the low energy limit on the world volume of a D_{p+3} brane in the appropriate type II string theory. Since this theory requires a UV completion, it is consistent to work in $p+4$ dimensions in the framework of an effective field theory, and assuming smallness of $E^2\theta$, where E is the characteristic energy scale in the problem (i.e. the typical value of momenta of on-shell particles), we linearize the $*$ product to get a combination

$$\phi_1 * \phi_2(x) = \phi_1 \phi_2(x) + i \frac{1}{2} \theta^{\mu\nu} \frac{\partial}{\partial x^\mu} \phi_1 \frac{\partial}{\partial x^\nu} \phi_2 + \dots \quad (3)$$

We are working in the limit when both θ and E^2 are sufficiently small so that M_{string}^2 is smaller than $1/(\theta^2 E^2)$. At the same time the loop integrals are cut off before the regime relevant for IR/UV sets in and our effective theory is consistent. Specializing the second term in this expansion to the case $p=1$, we immediately observe the presence of $\partial/\partial y$ derivative along the fifth dimension. Since at this point the theory we are discussing is an ordinary field theory deformed by a finite number of higher dimensional operators, we avoid the restrictions [26] on the momentum in the noncompact directions and can take the fifth dimension to be compact with the radius smaller than 10^{-17} cm so that the Kaluza-Klein modes of fermions, gauge bosons and scalars are heavy enough to avoid particle physics constraints. Our goal is to integrate out these heavy states and obtain the low-energy effective action for the zero-level Kaluza-Klein (KK) modes. Lowest energy modes do not have any y dependence. Therefore, $\theta_{\mu 5}$ may only appear in the loop-induced amplitudes with excited KK modes inside. Moreover, since the term we are interested in contains $i\partial/\partial y = p_5$ in one of the vertices, we have to find an amplitude that would contain an odd function of p_5 in the propagators because otherwise the loop amplitude will vanish upon summation of positive and negative p_5 modes. An odd function of p_5 may come only from the fermion propagators. Therefore, in order to get a nonvanishing result linear in $\theta_{\mu M}$ we have to allow fermions to propagate in a full five-dimensional space.

As a warm-up exercise we consider a two-loop-generated $\bar{\psi} \sigma_{MN} \theta^{MN} \psi$ amplitude in the NC scalar-fermion theory with Yukawa interaction $\lambda \bar{\psi} \psi \phi + i\lambda \theta_{MN} \bar{\psi} \partial_N \psi \partial_M \phi$. This calculation (similar to the 4D calculation performed in Ref. [6]) amounts to computing the contribution of KK modes to the self-energy of 4D fermions. The $\mu 5$ component of θ couples to $\bar{\psi} \sigma_{\mu, N=5} \psi = -\bar{\psi} \gamma_\mu \gamma_5 \psi$, a *CPT*-odd operator in 4D. Taking into account the contribution of the first excited KK level, we arrive at

$$\theta_{\mu 5} (\bar{\psi} \gamma_\mu \gamma_5 \psi) (\lambda^4 m / 32 \pi^4) M^2 \ln(\Lambda_{UV}^2 / M^2). \quad (4)$$

In this formula m and M are the masses of the zeroth and first KK levels and Λ_{UV} is an UV cutoff. The summation over the KK tower diverges as N^4 . Thus we would have to cut it at some value N_{max}^4 which exactly corresponds to an initial Λ_{UV}^4 divergence of the 5D two-loop integral. In a supersymmetric four-dimensional theory there can be a natural scale for the ultraviolet cutoff related to the soft breaking masses, $\Lambda_{UV} \sim m_{soft}$. We base this argument on the similarity of the $\theta_{MN} \bar{\psi} \sigma_{MN} \psi$ operator with the fermion anomalous magnetic moment which is not supersymmetrizable and thus must vanish in the exact super symmetric (SUSY) limit [17]. We believe, however, that the explicit calculation of $\theta_{MN} \bar{\psi} \sigma_{MN} \psi$ interaction in softly broken supersymmetric theory is needed to clarify this matter. To stay on the conservative side, we shall assume that the cutoff is not higher than few hundred GeV. Having the answer (4) at hand, one can try to determine the level of sensitivity to $\theta_{\mu 5}$ of different *CPT* and Lorentz violation searching experiments [5, 18–21] by identifying ϕ with the Higgs boson and λ with the light fermion Yukawa

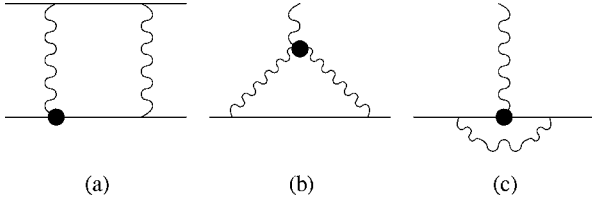


FIG. 1. One-loop diagrams in the 5D non-commutative $U(1)$ theory, which generate CPT noninvariant $\text{dim}=6$ operators in four dimensions. The heavy dot represents the interaction term with $\theta_{\mu 5}$ which can be placed at any vertex in (a) noncommutative box diagram + photon permutation, (b) noncommutative “bosonic” penguin. This diagram does not exist in a normal, commutative $U(1)$ theory. (c) Noncommutative “fermionic” penguin.

coupling. Clearly, one would have a serious suppression because all the relevant Yukawa couplings are quite small.

Thus, we are bound to explore gauge theories and limit our discussion to the case of the 5D noncommutative $U(1)$. One would naively guess that with the use of the noncommutative QCD in the bulk, the effective interaction is going to be at least two orders of magnitude stronger, simply because $\alpha_s \gg \alpha$. In fact QED and QCD may produce comparable results as it is known that multiple KK states accelerate the renormalization group running and already after few thresholds are taken into account, $\alpha_s \simeq \alpha$ [22].

First, we note that $m\theta_{MN}\bar{\psi}\sigma_{MN}\psi$ is not generated at one-loop level. In what follows we restrict our calculation to the one-loop level and compute the effective dimension-6 operators. Clearly, the use of higher dimensional operator will generically be suppressed by, say, $\Lambda_{\text{had}}^2/\Lambda_{UV}^2$ compared to the leading dimension operator, $m\bar{\psi}\sigma_{MN}\psi$. In our case, this suppression is not going to be dramatic since we choose a low value for the cutoff.

The relevant set of diagrams, Fig. 1, contains boxes, “bosonic” and “fermionic” penguins with all possible insertions of the θ -dependent vertices. We follow the linearized version of the Feynman rules for the noncommutative $U(1)$ theory given, for example, in [23]. As in the previous example, loops contain heavy KK modes. We simplify our calculation by working in zeroth order in the mass of the external fermions. Thus, in the box diagrams we can neglect all external momenta after which it immediately vanishes, as there is only the loop momentum to be contracted with θ_{MN} . The diagrams 1(b) and 1(c) produce the same operator. Taking into account the first KK mode propagating inside the loop, we get

$$\mathcal{L}_{1b+1c} = \frac{5e_i^2}{48\pi^2} \left[1 + \frac{9}{10} \frac{e_i}{e} \right] \theta_{\mu 5} (\bar{\psi}_i i \sigma^{\alpha\beta} \gamma_5 \psi_i) \partial_\mu F_{\alpha\beta}, \quad (5)$$

which turns out to be independent of the KK mass M . The summation over the whole tower is linearly divergent and gives N_{max} . In the square brackets of Eq. (5), 1 is the contribution of diagram 1(b) and $(9e_i)/(10e)$ is of 1(c).

The interaction (5) obviously breaks CPT . In addition to the electronic-dipole-moment-(EDM) like interaction, $F_{\alpha\beta} (\bar{\psi}_i i \sigma^{\alpha\beta} \gamma_5 \psi_i)$, Eq. (5) has an extra derivative ∂_μ which

changes sign under either P or T transformation. In order to obtain phenomenological consequences of this interaction, we specialize the result (5) to the case of the light quarks. For the matrix elements Δq of $\bar{q}\sigma_{\mu\nu}\gamma_5 q$ operators over a nucleon, we use the results of lattice [24] and QCD sum rule [25] calculations: $\Delta u_p = \Delta d_n \simeq 0.8$ and $\Delta u_n = \Delta d_p \simeq -0.2$. Thus we get

$$V = \frac{5e^3 N_{\text{max}}}{48\pi^2} \partial_\mu F_{\alpha\beta} \theta_{\mu 5} (\kappa_p \bar{p} i \sigma_{\alpha\beta} \gamma_5 p + \kappa_n \bar{n} i \sigma_{\alpha\beta} \gamma_5 n) \quad (6)$$

where

$$\kappa_p \simeq 0.11, \quad \kappa_n \simeq 0.08. \quad (7)$$

The use of the constituent quark model would produce 50% larger results.

One can treat $\partial_\mu F_{\alpha\beta}$ in Eq. (6) in two different ways. The most straightforward approach would be to try to estimate the photon loop nucleon self-energy diagram with one of the vertices given by Eq. (6). One can easily check that the Lorentz structure of such a diagram will be proportional to $\theta_{\mu 5} \bar{N} \gamma_\mu \gamma_5 N$. As to the numerical result of ultraviolet divergent integration, we cannot estimate it better than $O(e \times \text{loop factor} \times \Lambda^3)$ where Λ can be anywhere between m_π and m_ρ . Such a result could be considered as an order-of-magnitude estimate at best.

We prefer to use a different method and directly calculate the nuclear matrix element of interaction (6). We exploit the fact that the gradient of the electric field inside a large nucleus of atomic number Z and atomic mass A is approximately constant,

$$\partial_i F_{0j} \simeq \delta_{ij} Z e / R^3 \simeq \delta_{ij} e (Z/A) \text{fm}^{-3}. \quad (8)$$

For a nonrelativistic nucleon, inside the nucleus, V reduces to the product of the nucleon spin operator and θ_{i5} . The wave function of an external nucleon is concentrated mainly inside the nuclear radius R . Therefore, the nuclear matrix element reduces to a trivial angular part. Assuming one valence nucleon with orbital momentum L above closed shells, we arrive at the final form of the interaction between the nuclear spin I and the external vector θ_{i5} :

$$V = \frac{20}{3} (Z/A) N_{\text{max}} \alpha^2 \kappa_{p(n)} a_{LI} (\vec{I} \cdot \vec{\theta}) \text{fm}^{-3}. \quad (9)$$

Here $a_{LI} = \langle \vec{S} \cdot \vec{I} \rangle_{LI}$ is a trivial combination of $I(I+1)$ and $L(L+1)$, and $a = 1$ for $L = 0$.

The two most sensitive experiments, Ref. [5] and Ref. [20], use ^{199}Hg and ^{129}Xe whose spins are carried by external neutrons. Choosing $N_{\text{max}} = 1$ and comparing the size of V with the experimental accuracy of Ref. [20], $|V| < 2\pi \times 100 \text{ nHz}$, we deduce the level of sensitivity to the presence of mixed noncommutativity:

$$|\theta_{i5}| \lesssim (5 \times 10^{11} \text{ GeV})^{-2}. \quad (10)$$

Interestingly enough, we did not gain anything in terms of Λ_{NC} compared to our previous limit on θ_{ij} [4], even though

in addition to Lorentz symmetry the effective interactions (4) and (5) violate *CPT*. This is because in the case of mixed noncommutativity, we had to resort to a loop level and used low energy (small) value of α .

Do the results (4) and (5) mean that any five-dimensional model with $\theta_{\mu 5} \neq 0$ would violate *CPT* in the low-energy regime? The answer is no, as we can easily construct a counterexample. Indeed, let us consider a model where all fermions stay confined to a 3+1 domain wall. Then the parity along the extra dimension, $y \rightarrow -y$, will forbid any odd powers of coupling of $\theta_{\mu 5}$ to the four-dimensional fermions. Another obvious construction which helps to get rid of *CPT* violation at low energies is the orbifold projection in 5 dimensions. The same argument of exact parity in y coordinate will prohibit Eq. (6). As the result of orbifolding and/or fermion localization, only quadratic couplings in $\theta_{\mu 5}$ could be generated, thus relaxing experimental sensitivity to $\theta_{\mu 5}$. The most interesting phenomenological possibility would be a complete localization of the standard model degrees of freedom on a 3+1 domain wall with singlet neutrinos propagating in the bulk, in which case *CPT* violation would naturally occur in the leptonic sector.

III. CONCLUSIONS

The phenomenological consequences of the *CPT* violation are well understood, but the explicit models which break *CPT* are hard to find. In this paper we have demonstrated that the violation of the four-dimensional *CPT* invariance is

possible in the presence of mixed noncommutativity $\theta_{\mu 5}$. In particular, this happens when fermions are allowed to propagate in the five-dimensional bulk. We have shown that the Yukawa or gauge interaction of these fermions generate an effective four-dimensional $\theta_{\mu 5} \bar{\psi} \gamma_\mu \gamma_5 \psi$ as well as higher dimensional *CPT*-violating operators. Of course, an important ingredient in this picture is an indefinite parity for a fermion propagating in the five dimensional space.

Curiously enough, the tightest constraints in the case of $\theta_{\mu 5}$ do not arise from truly *CPT*-violation oriented experiments. We exploit the Lorentz noninvariance of this background and estimate the level of sensitivity of the clock comparison experiments [5] and [20] to the noncommutative scale as $(\theta_{\mu 5})^{-1/2} \sim 5 \times 10^{11}$ GeV in the case when quarks are allowed to propagate in the bulk.

An interesting feature of our result is the nondecoupling of heavy KK modes due to the ultraviolet enhancement brought by θ_{MN} . This behavior may change in the noncommutative SUSY theories which phenomenological consequences obviously deserve more careful analysis.

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